Exercise 10

(a) Find the slope of the tangent to the curve $y = 1/\sqrt{x}$ at the point where $x = a$.

(b) Find equations of the tangent lines at the points $(1, 1)$ and $\left(4, \frac{1}{2}\right)$.

(c) Graph the curve and both tangents on a common screen.

Solution

Part (a)

Start by finding the slope of the tangent line to the curve at $x = a$.

$$
\begin{aligned}
m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} &= \lim_{x \to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x - a} \\
&= \lim_{x \to a} \frac{\frac{\sqrt{a}}{\sqrt{a}} \cdot \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{x}}{\sqrt{x}}}{x - a} \\
&= \lim_{x \to a} \frac{\frac{\sqrt{a} - \sqrt{x}}{\sqrt{a}\sqrt{x}}}{x - a} \\
&= \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{a}\sqrt{x}(x - a)} \\
&= \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{a}\sqrt{x}(\sqrt{x} + \sqrt{a})(\sqrt{x} - \sqrt{a})} \\
&= \lim_{x \to a} \frac{-\cancel{(\sqrt{x} - \sqrt{a})}}{\sqrt{a}\sqrt{x}(\sqrt{x} + \sqrt{a})\cancel{(\sqrt{x} - \sqrt{a})}} \\
&= \lim_{x \to a} \frac{-1}{\sqrt{a}\sqrt{x}(\sqrt{x} + \sqrt{a})} \\
&= \frac{-1}{\sqrt{a}\sqrt{a}(\sqrt{a} + \sqrt{a})} \\
&= \frac{-1}{a(2\sqrt{a})} \\
&= -\frac{1}{2a^{3/2}}
\end{aligned}
$$

Part (b)

For the point $(1, 1)$, the slope is

$$
m = -\frac{1}{2(1)^{3/2}} = -\frac{1}{2}.
$$

The equation of the line is then

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}.$$

For the point $\left(4, \frac{1}{2}\right)$, the slope is

$$m = -\frac{1}{2(4)^{3/2}} = -\frac{1}{2(2)^3} = -\frac{1}{16}.$$

The equation of the line is then

$$y - \frac{1}{2} = -\frac{1}{16}(x - 4)$$

$$y - \frac{1}{2} = -\frac{1}{16}x + \frac{1}{4}$$

$$y = -\frac{1}{16}x + \frac{3}{4}.$$

Part (c)

Below is a graph of $y = 1/\sqrt{x}$ versus x along with the tangent lines at $x = 1$ and $x = 4$.